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FIRST LESSONS IN DEMONSTRATIVE GEOMETRY.

It is considered good pedagogy to make the approach to any new subject by means of related subjects of more elementary character. The first few lessons in Geometry need furnish no exception to the above rule. From his work in mensurational arithmetic, the pupil has a fair conception of areas of rectangles, triangles and circles, and volumes of some of the solids. From his experiences in everyday life, he has some idea of the fundamental meaning of the terms: point, straight line, angle, right angle, etc. From Algebra he has acquired the ability to choose symbols wisely and to use them skilfully, and has discovered how valuable they are in shortening operations. All of this fund of information may be used to great advantage by the careful teacher in the introductory lessons of Geometry.

The first class period should be devoted to the acquiring of the more common terms of the new subject. The technical words needed to discuss solids, surfaces, straight and curved lines, angles and angular measurement may well be introduced at the very beginning. A chalk box, the room in which the class works, or any other rectangular solid will furnish sufficient illustration. It is not wise to require too rigid definitions during the first few days. Accuracy of statement and clearness of language will come gradually after the beginner realizes the need of such accuracy. Freedom of discussion is more desirable at this point than rigidity of definitions. By skilful questioning the teacher should be able to gather from the different members of the class some good expressions of ideas that may be used a while at least for definitions.

For the next lesson the student should begin the solution of a large variety of exercises and questions dealing with the new language he has acquired and especially with the numbers of degrees in complementary and supplementary angles, adjacent angles, angles around a point and about a point on one side of a line. The use of Algebra may enter here and the teacher has a splendid opportunity to review simple equations and give the class a clear idea of many of the terms of Geometry at the same time. This work with a review of the terms already learned may be continued into the third or fourth recitation.

The next thing to be taken up is the work in construction. The tools of Geometry are the straight-edge and compasses and their use is limited to the well known postulates. Early in the course the student should know positively that tools other than these are used for experimental purposes only, as well as the use of these two tools beyond the limits of the postulates. There are many construction exercises the pupil can do at once. He should spend some time comparing lengths of given line-segments, constructing line-segments equal to the sum or difference of given segments, and constructing triangles whose sides are of given lengths or equal to the sides of a given triangle.

The protractor is next to be introduced and considerable experimental work should be done with it. The student should learn to use it in drawing angles of a given number of degrees and in measuring given angles. He should become familiar with several propositions of Geometry by means of its use and in other experimental ways; such as the theorems giving the sum of the angles of a triangle, the sum of the angles of a quadrilateral, the relative values of vertical angles, and the value of each angle of an equilateral triangle. It must be kept before his mind, however, that he has not proved any of these facts. That is, if he has tested five triangles, and finds that the sum of the angles of each is 180 degrees, he has not proved it true of every triangle. The use of the protractor need not be continued beyond the first few lessons. It is not, strictly speaking, a tool of geometry but is a valuable aid in developing a clear conception of the values of angles. A good student of Geometry should be able to estimate within a few degrees the value of any angle drawn on the blackboard. The interest aroused by the use of the tools of Geometry in construction and experiment may serve to awaken his powers of invention. There is in each child's mind an eagerness to find out things for himself. This desire is pretty well obliterated by our methods of feeding him information in huge doses without giving him the "why" of the information. Let us stop all this in Geometry and give him the satisfaction of deriving his own facts. Truths discovered by the pupil through his own efforts, even though hours of time are required, have an influence that is far-reaching and life-lasting.

Reasonable attention should be given the axioms and a few of the remaining definitions. Also a number of preliminary theorems should be proved informally in class before the formal proofs are begun. Not all the terms that are to be used in Geometry are to be defined during the first few lessons but only those that are immediately necessary. In fact it is very easy to overdo the work with the definitions. If the teacher takes a few minutes each recitation for a review of the meanings of the more important terms used, each pupil will soon have a valuable mathematical vocabulary without the apparent ill effects of too much memorizing.

As soon as the class is ready to begin the proofs of the theorems, it is a wise plan for the teacher to explain the meaning of a proof. Probably many of the failures in Geometry are caused by a misunderstanding of this very thing. Many teachers of this subject would be amazed if they could know what passes in the pupil's mind during the first few lessons of formal proofs. It requires much patience and considerable time for the teacher to get all the members of the class to realize that a completed proof is a chain of facts leading directly from a definite hypothesis to a definite conclusion. Each of these facts must be defended by a reason selected from one of the following geometrical sources: (1) the axioms, (2) the definitions, (3) the hypothesis of the theorems under consideration, and (4) the theorems and corollaries already proved.

There are various ways of accomplishing the desired results mentioned above. One is to make an exhaustive study of one of the earlier proofs that goes directly from the hypothesis to the conclusion in not more than five or six "steps." The class will be interested in studying how one step with its reason leads to the next and that to the next till the conclusion is the final step in the argument.

There are further suggestions which might be made for the younger teacher:

(*a*) Spend a part of a period in asking the members of the class to give the hypothesis and conclusion of several simple theorems.

(*b*) Require that all reasons, for a time at least, be given in full and complete sentences. Careless and slipshod English has no place in a course in Geometry.

(*c*) Compare the solution of a simple linear equation with the proof of a theorem. Notice particularly that each step in the solution may be defended by one or more axioms.

(*d*) Outline the proof of a theorem into four definite parts: (1) the exact statement of the theorem to be proved, (2) the "Given" clause which describes the figure and applies the hypothesis, (3) the "To prove" clause which explains what is to be proved, and (4) the proof.

Before this paper is brought to a close, it is necessary to explain that it has been written with those pupils in mind who take up the study of Geometry as a new subject after a year of elementary Algebra. There is no intention on the part of the authors to argue that that order of studies is the best plan. Because the majority of schools offer the subjects in that order is sufficient reason for the suggestions in the preceding paragraphs. For those schools that offer any form of elementary Geometry in the eighth or ninth year, or that teach any kind of correlated mathematics, these suggestions should of course be modified.

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